Dealing with the Coronavirus: Some Back of the Envelope Calculations*

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Abstract

This paper presents a simple model to quatify the welfare consequences of alternative ways to deal with the coronavirus.

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1 Introduction

In December 2019, an outbreak of a strain of coronavirus (named COVID-19) was reported in Wuhan, China. Since then, as of March 24, 2020, almost 420 thousand cases of infected individuals have been reported in 169 countries, with almost 19 thousand deaths; and more than 107 thousand recovered.

Its speed of proliferation and global scale has had devastating consequences for the economies and the well-being of individuals.

In response, governments have implemented different policies, aimed at containing the spread of the virus and "flatten(ing) the curve" of infected individuals, so that the medical facilities to treat sick persons are not overwhelmed.[1]

There is variance in terms of policies implemented, with some governments resorting to forced quarantines and lockdowns. Individuals have also responded heterogenous responses to "social distancing" in different countries. On the one extreme, a complete lockdown would slow the spread of COVID-19, but could have catastrophic consequences on the economy and well-being of the individuals. On the other extreme, no action to prevent the spread of the virus could also have devastating consequences in terms of human lives.

This note provides a simple framework with which to analyze the relevant trade-offs that this outbreak posits. It is organized as follows: Section 2 presents a stylized model to characterize the dynamics of the epidemic, its economic consequences, and the way in which both features reflect on the welfare of individuals. Section 3 provides some numerical exercises that can be used to assess the impact of different strategies aimed at containing the spread of the virus. Finally, Section 4 concludes.

2 A very simple model

This section presents simple and stylized models to characterize the dynamics of the epidemic and its economic consequences.[2]

[1] A Washington Post article by Harry Stevens has become viral, presenting simulations with the evolution of an hypothetical epidemic under different social distancing scenarios (https://www.washingtonpost.com/graphics/2020/world/corona-simulator/).

[2] I have recently had access to a paper by Eichenbaum et al. (2020) that has a similar structure. I will discuss the differences as I go along.

2.1 The epidemic

Following Allen (1994) and Hetchcote (1989) consider the *SIR* epidemic model,[3] characterized by the following difference equations:

$$S_t = S_{t-1}\left(1 - \frac{\alpha}{N} I_{t-1}\right) \quad (1)$$

$$I_t = I_{t-1}\left(1 - \gamma + \frac{\alpha}{N} S_{t-1}\right) \quad (2)$$

$$R_t = R_{t-1} + \gamma I_{t-1}, \quad (3)$$

where S_t is the susceptible (but still healthy) population at date t, I_t is the infected (and not yet recovered) population, R_t is the number of recovered individuals, $N = S_t + I_t + R_t$ is the total (constant) population, α is the contact rate,[4] and γ is the relative removal rate.[5]

Given the initial conditions $S_0 > 0, I_0 > 0, R_0 \geq 0$, equations (1)-(3) characterize the dynamics of the components of the total population. Furthermore, solutions to this discrete system are positive for all initial conditions, if and only if (Allen, 1994):

$$\max\{\alpha, \gamma\} \leq 1.$$

Finally, define the reproduction rate as:

$$\mathcal{R} = \frac{S_0 \alpha}{N \gamma} \quad (4)$$

whose value determines the global behavior of the model. If $\mathcal{R} \leq 1$ there is no epidemic as the infective class always decreases. On the other hand, if $\mathcal{R} > 1$, the infective class initially increases and eventually decreases; while there always remain some susceptible after the epidemic has ended.

Thus, apart from the initial conditions, the key (and only) parameters of this model are α and γ.

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[3] The SIR model divides the population in three groups: Susceptible, Infected, and Recovered. The model can be extended to include vital dynamics (births and deaths).

[4] Defined as the average number of individuals with whom an infectious individual makes contact to pass the infection.

[5] Defined as the probability that one infected will be removed from the infection process, which is equivalent to the reciprocal of the average duration of the infection.

2.2 The economy

Containment policies have economic consequences in several dimensions. For starters, infectious individuals suffer deleterious consequences on their health and productivity. Furthermore, recovered individuals may present sequels that also affect their productivity (at least for a while). Finally, policies like lockdowns and quarantines also have a negative effect on the productivity of the individuals.[6]

Thus, we define the output of individuals as dependent on their health state and the containment policies that can be, indirectly, linked to the contact and relative removal rates (α, γ).

Thus, the total output Y generated in period t is given by:

$$Y_t = a(\alpha, \gamma) S_t + b(\alpha, \gamma) I_t + c(\alpha, \gamma) R_t, \tag{5}$$

where a, b, c depend on the values of α, γ; and satisfy:

$$1 \geq a \geq c > b \geq 0, \quad \forall \alpha, \gamma.$$

That is, susceptible individuals are at least as productive as recovered individuals, who, in turn, are more productive than infectious individuals, irrespective of the contact or relative removal rates.

Finally, in case of no outbreak ($I_0 = 0$), we set $a = 1$, where every individual is assumed to produce one unit of output, and $Y_t = N$.

2.3 Welfare

To evaluate the welfare consequences of alternative containment policies, consider a representative agent whose utility depends on his consumption (proxied by his income) and health status.[7] Concretely,

$$U(\alpha, \gamma, I_0, S_0) = \sum_{t=0}^{T} \beta^t \left[\left(1 - \frac{I_t}{N}\right) \ln\left(\frac{Y_t}{N}\right) \right], \tag{6}$$

where β is the discount factor and T is the final period for which a proposed containment policy is to be evaluated. In this case, the fraction of currently infected acts as a "bad" an decreases utility.

[6] As a large fraction of jobs can not be done as productively (or at all) from home.

[7] See for example Viscusi and Evans (1990) and Viscusi (2019) for alternative ways in which the health status enters the utility function.

As a benchmark, the case of no outbreak ($I_0 = 0$) trivially renders $Y_t = S_t = N$ and $U(I_0 = 0) = 0$.

Another approach to measure the welfare of the representative individual is to consider the utilities of consumption weighted by the fraction of population in each state, that is:[8]

$$V(\alpha, \gamma, I_0, S_0) = \sum_{t=0}^{T} \beta^t \left[\frac{S_t}{N} \ln(a) + \frac{R_t}{N} \ln(b) + \frac{I_t}{N} \ln(c) \right], \tag{7}$$

where, as indicated in (5) and discussed below, a, b, c depend on the containment strategies.

As in the case of (6), in the benchmark of no outbreak ($I_t = 0$), $Y_t = S_t = N$ and $V(I_0 = 0) = 0$.

3 Results

We interpret that alternative containment policies can be summarized in different values of α, γ. For example, stricter rules that prevent or ameliorate human contact would lead to lower values of α. On the other hand, without a vaccine or other medical breakthroughs, slowing the spread of the infection can be understood as a way to increase γ. That is, for a given number of hospitals, a fixed supply of ICU beds and ventilators, and more people infected, more infected individuals would lead to a slower average recovery (decreasing γ), and, tragically, more deaths.

The idea of "flatten(ing) the curve" can be achieved by decreasing α, increasing γ, or using any combination of both parameters that ultimately decreases the reproduction rate, $\mathcal{R}$, in (4). Decreasing $\mathcal{R}$ by changing α and/or γ, will adversely affect the productivities (and incomes) of the individuals. There lays the trade-off.

We consider the implications of having different configurations of parameters on the spread of the virus, incomes, and welfare of the population. To do so, we:

- Set the size of the population N, and the initial values of infected and susceptible individuals (I_0 and S_0 respectively).

[8] This could be interpreted as a type of expected utility.

- Choose a configuration of parameters α and γ.
- State the effects of α and γ on the values of a, b, c.
- Simulate the trajectories of $\{S_t, I_t, R_t, Y_t\}_{t=0}^{T}$ given the values of α, γ, a, b, c, I_0 and S_0.
- Given a value of β, use the trajectories obtained in the previous step to compute the discounted utility, suing either (6) or (7).

These same steps can be conducted to obtain the trajectories of the variables and the discounted utility for a different configuration of parameters and assess its economic and epidemiological consequences.

A natural way to compare alternative configurations is to use a tool popularized by Lucas (1987) to asses the consequences on welfare of alternative policies. Simply put, the question that we would like to ask is "how much (as a percentage of income) should we transfer an individual to make him as well-off as if there was no outbreak of coronavirus?".

Depending on the utility chosen, we solve for the value of τ that satisfies:

$$\begin{aligned}\sum_{t=0}^{T}\beta^t\left[\left(1-\frac{I_t}{N}\right)\ln\left(\frac{(1+\tau_U)\,Y_t}{N}\right)\right] &= 0 = U\left(I_0=0\right), \\ \sum_{t=0}^{T}\beta^t\left[\ln\left(1+\tau_V\right)+\frac{S_t}{N}\ln\left(a\right)+\frac{R_t}{N}\ln\left(b\right)+\frac{I_t}{N}\ln\left(c\right)\right] &= 0 = V\left(I_0=0\right).\end{aligned}$$

The solutions are:

$$\begin{aligned}\tau_U &= \exp\left(-U\left(\alpha,\gamma,I_0,S_0\right)/\left[\sum_{t=0}^{T}\beta^t\left(1-\tfrac{I_t}{N}\right)\right]\right)-1 \\ \tau_V &= \exp\left(-\left(1-\beta\right)V\left(\alpha,\gamma,I_0,S_0\right)/\left(1-\beta^{T+1}\right)\right)-1\end{aligned}. \qquad (8)$$

As an exploratory exercise, next I present the solutions for this model using an hypothetical example. To do so, I consider a daily model with $T = 365$ and set $\beta = 0.99$. That is, I consider the effects until one year after the outbreak began. I set total population to $N = 5$ million, $I_0 = 2$ initially infected individuals, and $R_0 = 0$. The other parameters are set according to different scenarios (Table 1).

Scenario 0 intends to approximate the case in which there are relatively high interactions between individuals. This is captured by setting the value of $\mathcal{R}$ to 2.5, which is approximately the value that the (abundant, but still imprecise) literature on the subject uses based on Wuhan (Imperial College, 2020).[9] We also set the value of γ to 1/14 which means that the average time of recovery, once infected, is of 14 days. This means that the value of α is set to 2.5γ. Finally, we assume that in this environment, susceptible individuals conduct "business as usual" and their productivity is the same as in the case of no outbreak. We consider that the productivity of the recovered individuals is 1% lower, assuming there are minor side-effects after recovery. Finally, we assume that the infected have a productivity that is 50% lower that the (still healthy) susceptible individuals.

Table 1: Parameters under different scenarios

Parameter	Scenario 0	Scenario 1	Scenario 2
α	2.5/14	2/12	1.5/10
γ	1/14	1/12	1/10
a	1	0.9	0.8
b	0.99	0.891	0.792
c	0.5	0.45	0.4

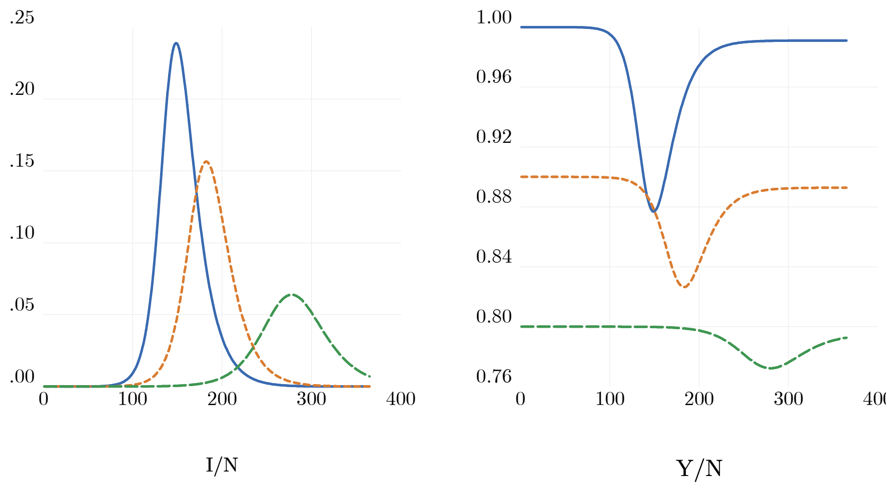
Note: Scenario 0,1,2 = High, Medium, Low interaction.

Scenarios 1 and 2 consider the cases in which more aggressive approaches are taken, allowing for medium and low interactions between individuals. In Scenario 1, $\mathcal{R}$ is set to 2 (which is in the lower spectrum of values estimated in early models) and γ is set to 1/12. In Scenario 2, $\mathcal{R}$ is set to 1.5 and γ is set to 1/10. In both cases, we intend to capture the idea that, with less people infected, medical services can be better used to speed up recovery. Finally, we assume that the measures taken to limit interactions have a deleterious effect on the productivity of the individuals, with a productivity 10% (20%) lower than in Scenario 0 when considering Scenario 1 (2). Again, this feature intends to capture than lockdowns, limited mobility, closing of factories and offices, and other measures affect productivity and incomes.[10]

Figure 1 displays the results for the daily trajectory of the fraction of

[9] This value is greater than the ones used by Eichenbaum et al. (2020), but is more in

Figure 1: Results for different configurations of parameters



Note: Blue = Scenario 0. Red = Scenario 1. Green = Scenario 2.

the population that is infected (first panel) and per capita income (second panel). The first panel shows the (by now, viral) characterization of "flatten the curve", that reduces the reproduction rate. In Scenario 0 the peak in the number of infected individuals occurs 149 days after the outbreak, comprising 23.8% of the population. In Scenario 1 the peak of infected individuals occurs 182 days after the outbreak, comprising 15.6% of the population. Finally, the peak of infected individuals in Scenario 2 occurs at date 278 after the outbreak, comprising "only" 6.4% of the population.

The good news of limiting interactions stop there, as output, as measured by (5) behave in the opposite direction. Scenarios 1 and 2 severely depress the economy, as compared to Scenario 0. Even though more aggressive measures to limit interactions delay the peak in the number of infected individuals, the economy contracts with the intervention.

Table 2 quantifies the welfare costs of each scenario compared to the benchmark case of no outbreak using (8). Scenario 0 (which allows for high interactions) results in a welfare loss of the equivalent of between 1.45% and

line with the progression of the infections in Europe and the US.

[10] A GAUSS executable program where the user can modify all these parameters will be made available shortly.

Table 2: Welfare costs of different scenarios (% of consumption)

Parameter	Scenario 0	Scenario 1	Scenario 2
τ_U	1.45	12.05	25.26
τ_V	2.15	12.45	25.36

Note: Scenario 0,1,2 = High, Medium, Low interaction.

2.15% of contingent consumption, while Scenario 1 (2) brings a welfare loss of the equivalent of (approximately) 12% (25%) of consumption. These are huge numbers. Thus, in terms of welfare, Scenarios 1 and 2 would by several orders of magnitude costlier than Scenario 0, if welfare is measured by (6) or (7).

These results (crucially) depend on the assumptions that the reproduction rate $\mathcal{R}$ and productivity effects are constant, which would imply that the containment policies are in place for the whole period.[11] In practice, individuals and governments endogenously respond to changing conditions and affect the reproduction rate $\mathcal{R}$.[12] Furthermore, we are assuming that measures that affect the reproduction rate also imply effects (during the whole year) of the productivity of the workers. As conditions improve, restrictions may be endogenously lifted. Nevertheless, remember that with more stringent containment conditions, we delay the peak of the infections. In that case, individuals and governments may maintain or prolong containment conditions, further deteriorating the economy.

This simple model does not consider other policies that are discussed to ameliorate the effects of the epidemic.[13] However, when these measures include massive interventions in the economy that could prevent or delay

[11] This is a strong assumption. Although several governments are suggesting incorporating or extending containment policies for more days than initially proposed.

[12] An interesting discussion of this feature can be found on https://reason.com/video/dont-expect-millions-to-die-from-coronavirus-says-richard-epstein/

[13] Eichenbaum et al. (2020) consider cases in which there is space for an optimal containment policy. Their reasoning relies on the fact that in a competitive equilibrium, individuals do not fully internalize the negative externality of their actions on the infection rates. However, their optimal policies rely on the unrealistic assumption that the central planner knows perfectly well the preferences, technology and characteristics of the epidemic.

the signals that market forces require to reallocate resources,[14] these policies may produce more harm than good; as they could slow down even more the recovery of the economy.[15]

4 Concluding Remarks

The COVID-19 epidemic posits major and dramatic challenges for individuals and governments. This note presents a simple model that puts into evidence the non trivial trade-offs faced. Containment measures have costs and benefits.

As a believer in the division of labour, doctors and health care professionals are devoting (and risking) their lives to save individuals infected by the virus, epidemiologists are working on models that help to better understand the characteristics of the epidemic, scientists are working on developing treatments and vaccines, and economists should work on providing information that explains and (hopefully) quantifies the relevant trade-offs. More and better information should be used for taking more informed decisions and policies.

[14] For example, in the volume edited by Baldwin and Weder di Mauro (2020), several authors analyze and propose huge scale interventions.

[15] Cole and Ohanian (2004) discuss this feature in the case of the Great Recession.

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